Instructions: Try these problems from the textbook as practice (answers below).

Problems

3-16. Each of these extreme value problems has a solution with both a maximum value and a minimum value. Use Lagrange multipliers to find the extreme values of the function subject to the given constraint.

3. $f(x,y) = x^2 - y^2; x^2 + y^2 = 1$
5. $f(x,y) = xy; 4x^2 + y^2 = 8$
7. $f(x,y) = 2x^2 + 6y^2;\ x^2 + y^2 = 2$
9. $f(x,y,z) = 2x + 2y + z;\ x^2 + y^2 + z^2 = 9$
13. $f(x,y,z) = x^2 + y^2 + z^2;\ x^4 + y^4 + z^4 = 1$
15. $f(z,y,z,t) = z + y + z + t;\ x^2 + y^2 + z^2 + t^2 = 1$

25. Use Lagrange Multipliers to find the maximum value of $f(x,y) = e^{xy}$ subject to the constraint $x^3 + y^3 = 16$. Then show that f has no minimum value with that constraint.

27-29. Find the extreme values of f on the region described by the inequality.

27. $f(x,y) = x^2 + y^2 + 4x - 4y; x^2 + y^2 \leq 9$
29. $f(x,y) = e^{-xy}; x^2 + 4y^2 \leq 1$

39. Use Lagrange multipliers to prove that the rectangle with maximum area that has a given perimeter p is a square.

55. A grain silo is to be built by attaching a hemispherical roof and a flat floor onto a circular cylinder. Use Lagrange multipliers to show that for a total surface area S, the volume of the silo is maximized when the radius and height of the cylinder are equal.

Answers

3. Global minimum: -1 at $(0, \pm 1)$
   Global maximum: 1 at $(\pm 1, 0)$

5. Global minimum: -2 at $(1, -2), (-1, 2)$
   Global maximum: 2 at $(1, 2), (-1, -2)$

7. Global minimum: 4 at $(\pm\sqrt{2}, 0)$
   Global maximum: 12 at $(0, \pm\sqrt{2})$

9. Global minimum: $-9\sqrt{2}$ at $(-2\sqrt{2}, -2\sqrt{2}, -\sqrt{2})$
   Global maximum: $9\sqrt{2}$ at $(2\sqrt{2}, 2\sqrt{2}, \sqrt{2})$

13. Global minimum: $\sqrt{3}$
    Global maximum: 1

15. Global minimum: 2
    Global maximum: 2

25. Global maximum value is e^4

27. Global maximum: $9 + 12\sqrt{2}$ at $(3/\sqrt{2}, -3/\sqrt{2})$
    Global minimum: -8 at $(-2, 2)$

29. Global maximum: $e^{1/4}$
    Global minimum: $e^{-1/4}$

39. $\begin{cases} y = 2\lambda \\ x = 2\lambda \\ 2x + 2y = p \end{cases}$, so $x = y$.

55. $\begin{cases} rh + r^2 = (3r + h)\lambda \\ r^2 = 2r\lambda \\ 3r^2 + 2rh = S/\pi \end{cases}$. Manipulate to obtain $r = 2\lambda = h$.